Solutions:

① $r(t) = \langle 4\cos t, 4\sin t, 0\rangle$ (One possible parameterization)

$K(t) = \dfrac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$ $\quad r'(t) = \langle -4\sin t, 4\cos t, 0\rangle$

$r''(t) = \langle -4\cos t, -4\sin t, 0\rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -4\sin t & 4\cos t & 0 \\ -4\cos t & -4\sin t & 0 \end{vmatrix} = k\left(16\sin^2 + 16\cos^2\right) = 16\,k$$

$\|r'(t) \times r''(t)\| = 16$ $\quad \|r'(t)\| = \sqrt{4^2\sin^2 + 4^2\cos^2} = 4$

$$K(t) = \frac{16}{4^3} = \frac{1}{4}$$

② $r(t) = \langle 2+3t, 7t, 5-t\rangle$ (Guess: 0, it's a straight line)

$r'(t) = \langle 3, 7, -1\rangle$, $r''(t) = \langle 0, 0, 0\rangle$

$\Rightarrow r'(t) \times r''(t) = 0 \Rightarrow \boxed{K(t) = 0}$

a) $K(t) = \dfrac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$

$r'(t) = \langle -\sin t, \cos t, 2t\rangle$, $r''(t) = \langle -\cos t, -\sin t, 2\rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -\sin t & \cos t & 2t \\ -\cos t & -\sin t & 2 \end{vmatrix}$$

$$= i\left(2\cos + 2t\sin\right) - j\left(-2\sin + 2t\cos\right) + k\left(1\right)$$

$$\|r'(t) \times r''(t)\| = \sqrt{(2\cos + 2t\sin)^2 + (2\sin - 2t\cos)^2 + 1^2}$$

$$= \sqrt{4\cos^2 + \cancel{4t\cos\sin} + 4t^2\sin^2 + 4\sin^2 - \cancel{4t\cos\sin} + 4t^2\cos^2 + 1}$$

$$= \sqrt{4+4t^2+1} \qquad = \sqrt{4t^2+5}$$

$$\|r'(t)\| = \sqrt{\sin^2+\cos^2+4t^2} = \sqrt{4t^2+1}$$

$$K(t) = \frac{\sqrt{4t^2+5}}{(4t^2+1)^{3/2}}$$

b. $$K(t) = \frac{|x'(t)\,y''(t) - y'(t)\,x''(t)|}{(x'(t)^2 + y'(t)^2)^{3/2}}$$

$$x(t) = t^2 \qquad x'(t) = 2t \qquad x''(t) = 2$$

$$y(t) = t^3 \qquad y'(t) = 3t^2 \qquad y''(t) = 6t$$

$$K(t) = \frac{|12t^2 - 6t^2|}{(4t^2+9t^4)^{3/2}} = \frac{6t^2}{(t^2)^{3/2}(4+9t^2)^{3/2}} = \frac{6}{t(4+9t^2)^{3/2}}$$

c. $$K(x) = \frac{|f''(x)|}{(1+f'(x)^2)^{3/2}}$$

$$f(x) = e^x, \quad f'(x) = e^x, \quad f''(x) = e^x$$

$$K(x) = \frac{e^x}{(1+e^x)^{3/2}}$$

(4) $r(t) = \langle 0, t, t^2 \rangle \qquad r'(t) = \langle 0, 1, 2t \rangle \qquad \|r'(t)\| = \sqrt{4t^2+1}$

$$T(t) = \boxed{\left\langle 0, \frac{1}{\sqrt{4t^2+1}}, \frac{2t}{\sqrt{4t^2+1}} \right\rangle}$$

$$T'(t) = \left\langle 0, -\frac{1}{2}(4t^2+1)^{-3/2} \cdot 8t, \frac{2\sqrt{4t^2+1} - \cancel{2}t \cdot \cancel{\frac{1}{2}}(4t^2+1)^{-1/2} \cdot 8t}{4t^2+1} \right\rangle$$

$$= \left\langle 0, -(4t^2+1)^{-3/2} \cdot 4t, 2(4t^2+1)^{-1/2} - 8t^2(4t^2+1)^{-3/2} \right\rangle$$

$$= \left\langle 0, \frac{-4t}{(4t^2+1)^{3/2}}, \frac{2}{(4t^2+1)^{3/2}} \right\rangle$$

$$\|T'(t)\| = \sqrt{\frac{(4t)^2}{(4t^2+1)^3} + \frac{4}{(4t^2+1)^3}} = \frac{2\sqrt{4t^2+1}}{(4t^2+1)^{3/2}}$$

$$= \frac{2}{4t^2+1}$$

$$N(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{4t^2+1}{2}\left\langle 0, \frac{-4t}{(4t^2+1)^{3/2}}, \frac{2}{(4t^2+1)^{3/2}}\right\rangle$$

$$= \boxed{\left\langle 0, -\frac{2t}{\sqrt{4t^2+1}}, \frac{1}{\sqrt{4t^2+1}}\right\rangle}$$

$$B(t) = T \times N = \begin{vmatrix} i & j & k \\ 0 & 1/\sqrt{4t^2+1} & 2t/\sqrt{4t^2+1} \\ 0 & -2t/\sqrt{4t^2+1}, & 1/\sqrt{4t^2+1} \end{vmatrix}$$

$$= i\left(\frac{1}{4t^2+1} + \frac{4t^2}{4t^2+1}\right) - j\,(0) + k\,(0)$$

$$= \boxed{i} \quad \text{(Woah)}$$

$t=1$:

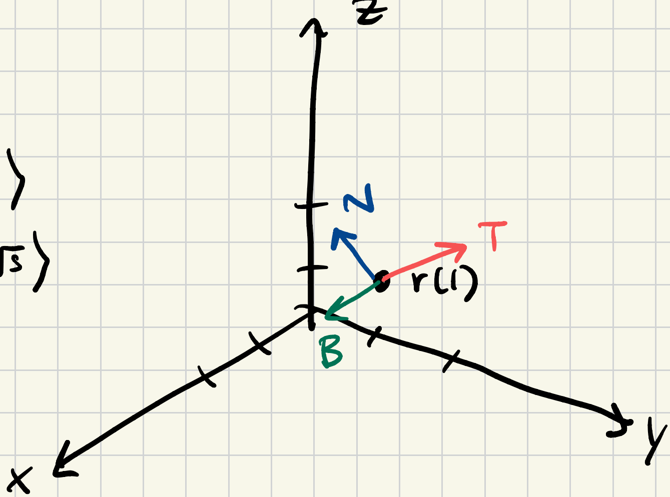
$$\frac{-4}{5^{3/2}}$$

$r(1) = \langle 0, 1, 1\rangle$

$T(1) = \langle 0, 1/\sqrt{5}, 2/\sqrt{5}\rangle$

$N(1) = \langle 0, -2/\sqrt{5}, 1/\sqrt{5}\rangle$

$B(1) = \langle 1, 0, 0\rangle$



(Swapped 3 & 4)

3 $N(t)$ for $r(t) = \langle R\cos t, R\sin t, 0\rangle$

$$r'(t) = \langle -R\sin t, R\cos t, 0\rangle$$

$$\|r'(t)\| = \sqrt{R^2\sin^2 + R^2\cos^2} = R$$

$$T(t) = \frac{1}{R}\langle -R\sin t, R\cos t, 0\rangle = \langle -\sin t, \cos t, 0\rangle$$

$$T'(t) = \langle -\cos t, -\sin t, 0\rangle, \quad \|T'(t)\| = 1$$

$$\Rightarrow \boxed{N(t) = \langle -\cos t, -\sin t, 0\rangle}$$

5 Decompose $a(t)$ into a_T and a_N components:

a) $r(t) = \langle 4-t, t+1, t^2\rangle$

b) $r(t) = \langle t, e^t, te^t\rangle$

5 a) $r(t) = \langle 4-t, t+1, t^2\rangle$

$$r'(t) = \langle -1, 1, 2t\rangle = v(t) \qquad \|v(t)\| = \sqrt{4t^2+2}$$

$$r''(t) = \langle 0, 0, 2\rangle = a(t) \qquad \|a(t)\| = 2$$

$$a_T = \frac{a\cdot v}{\|v\|} = \boxed{\frac{4t}{\sqrt{4t^2+2}}}$$

$$a_N = \sqrt{\|a\|^2 - a_T^2} = \sqrt{4 - \frac{16t^2}{4t^2+2}} = \sqrt{\frac{16t^2+8}{4t^2+2} - \frac{16t^2}{4t^2+2}}$$

$$= \boxed{\sqrt{\frac{8}{4t^2+2}}}$$

b) $r(t) = \langle t, e^t, te^t \rangle$

$e^{2t} + 2$

$e^t(t+1)$

$v(t) = r'(t) = \langle 1, e^t, e^t + te^t \rangle$

$a(t) = r''(t) = \langle 0, e^t, (t+2)e^t \rangle$

$$\|v(t)\| = \sqrt{1 + e^{2t} + e^{2t}(t+1)^2}$$

$$a_T = \frac{a \cdot v}{\|v\|} = \boxed{\frac{e^{2t} + (t+1)(t+2)e^{2t}}{\sqrt{1 + e^{2t} + e^{2t}(t+1)^2}}}$$

$$\|a(t)\| = \sqrt{e^{2t} + e^{2t}(t+2)^2}$$

$$a_N = \sqrt{\|a\|^2 - a_T^{\,2}}$$

$$= \sqrt{e^{2t} + e^{2t}(t+2)^2 - \frac{\left(e^{2t} + (t+1)(t+2)e^{2t}\right)^2}{1 + e^{2t} + e^{2t}(t+1)^2}}$$

$$= \sqrt{\frac{e^{2t} + e^{2t}(t+2)^2 + \cancel{e^{4t}} + e^{4t}(t+2)^2 + e^{4t}(t+1)^2 + \cancel{e^{4t}(t+1)^2(t+2)^2}}{1 + e^{2t} + e^{2t}(t+1)^2} - \frac{\cancel{e^{4t}} - 2e^{4t}(t+1)(t+2) - \cancel{(t+1)^2(t+2)^2 e^{4t}}}{1 + e^{2t} + e^{2t}(t+1)^2}}$$

$$= \boxed{e^t \sqrt{\frac{1 + e^{2t} + (t+2)^2}{1 + e^{2t} + e^{2t}(t+1)^2}}}$$